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MULTIREGIONÁLNA HOSPODÁRSKA DYNAMIKA S CENAMI PÔDY A AGLOMERÁCIOU

MULTI-REGIONAL ECONOMIC DYNAMICS WITH LAND PRICES AND AGGLOMERATION

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Cieľom štúdie je analýza dynamiky hodnoty pôdy a pozemkovej renty v spojitosti s medziregionálnou aglomeráciou. Ekonomika sa skladá z ľubovoľného počtu regiónov, každý s dvomi výrobnými sektormi. Hodnota pôdy a pozemkovej renty, akumulácia kapitálu a regionálne vybavenie sú endogénne. Ekonomika je postavená na predpokladoch maximalizácie zisku, maximalizácie užitočnosti a ideálnej konkurencie. Ukazujeme, že dynamiku národného hospodárstva regiónu *J* riadia diferenciálne rovnice *J+1*. Simulujeme model a demonštrujeme existenciu jedinečného rovnovážneho bodu. Komparatívnou dynamickou analýzou predstavujeme ako zmeny produktivity výrobných faktorov, parametrov regionálnej vybavenosti a tendencií spotrebovať priemyselné produkty, ako aj udržiavanie a užívanie obydlí ovplyvňujú všetky premenné.
Kľúčové slová: hodnota pôdy, medziregionálna nerovnosť, regionálna aglomerácia, akumulácia bohatstva, regionálna vybavenosť

The purpose of this study is to analyze dynamics of space-dependent land values and rents in tandem with interregional agglomeration. The economy consists of any number of regions and each region has two production sectors. Land values, land rents, capital accumulation and regional amenities are endogenous. The economy is built under assumptions of profit maximization, utility maximization, and perfect competition. We show that the dynamics of *J* region of national economy is controlled by *J+1* differential equations. We simulate the model and demonstrate the existence of a unique equilibrium point. We conduct comparative dynamic analysis to plot the effects on all the variables due to changes in the total factor productivities, regional amenity parameters, and the propensities to consume industrial goods, to save and to consume housing.
Keywords: land value, interregional inequality, regional agglomeration, wealth accumulation, amenity
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1 INTRODUCTION

Land value and its dynamics over time and space is a daily concern for and determinants. People from different parts of the work are buying and selling land as economic globalization is deepening. The basic economic mechanism of determination of land value is important for understanding changes of land values over time and space. There are many studies on house and land prices in recent years as surveyed by Cho (1996, p. 145), "During the past decade, the number of studies on intertemporal changes in house prices has increased rapidly because of wider availability of extensive micro-level data sets, improvements in modeling techniques, and expanded business applications." (e.g., Bryan and Colwell 1982, Clapp and Giaccotto 1994, Kok et al. 2014). Nevertheless, determination of land values over time and space has been almost completely neglected in the economic theories with microeconomic foundation. This study is to examine dynamic interdependence between economic growth and land value change over space. The issues under consideration are complicated as we need to explicitly introduce land into a growth model with capital accumulation. Economics still needs an analytical framework for analyzing these issues. It should be noted that some issues related to land are examined by the specific factor (SF) model or the Rivardo-Viner model, which is a variation of the Ricardian model proposed by Viner. The model was later formalized mathematically by Jones (1971, see also Samuelson, 1971). There are many studies on generalizing this model. The model is also referred to as the two-good, three-factor model. In this modelling framework, one factor is "specific" to a particular industry. The factor is immobile between sectors and is assumed to be stuck in the industry. It is interesting to allow some specific factors in our framework.

Although there are many studies on land values and economic growth, most of these studies are empirical. There are only a few formal growth models with endogenous land values. In his classical work On the Principles of Political Economy and Taxation of 1817, Ricardo tried to link wages, interest rate, and rent together in a compact theory. Ricardo distinguished between the three production factors, labor, capital, and land. He provided a theory to explain the functional income distribution of labor share, the capital, and the land rent share of total income. Ricardo (1821) pointed out: "The produce … is divided among three classes of the commodity, namely, the proprietor of land, the owners of the stock or capital necessary for its cultivation, and laborers by whose industry it is cultivated. But in different stages of the society, the proportions of the whole produce of the earth which will be allotted to each of these classes, under the names of rent, profits, and wages, will be essentially different; depending mainly on the actual fertility of the soil, on the accumulation of capital and population, and on the skill, ingenuity, and the instruments in agriculture." Nevertheless, Ricardo did not create a theory for determination of land value. After the publication of the Principles, many attempts have been made to extend or generalize the Ricardian system (see Barkai 1959, Pasinetti 1960, Brems 1970, Casarosa 1985, Morishima 1989). Nevertheless, what Ricardo (1821) observed long time ago is still valid to describe the current state of the literature: "To determine the laws which regulate this distribution, is the principal problem in Political Economy: much as the science has been improved by the writings of Turgot, Stuart, Smith, Say, Sismondi, and others, they afford very little satisfactory information respecting the natural course of rent, profit, and wages." In Ricardo's statement there is even no reference to land value (price). If land is owned by a single owner, it is not misleading to neglect determination of land value. Nevertheless, in modern economies land is owned by many people. As recently reviewed by Liu et al. (2011, p. 1), "Although it is widely accepted that house prices could have an important influence on macroeconomic fluctuations, quantitative studies in a general equilibrium framework have been scant." This study contributes to the literature by introducing endogenous land price into a multi-regional growth model recently proposed by Zhang (2018).

This study is based on the neoclassical growth theory. It includes endogenous wealth accumulation, land value, economic structure and amenity in modelling multi-regional growth. Following the neoclassical growth theory, we consider capital accumulation a main determinant of economic growth and development. In the literature of spatial economic growth only a few formal models include capital accumulation in modelling dynamic economic geography with rational assumptions of profit and utility maximization. Moreover, our approach differs from the so-called new economic geography which currently is a fast-growing research field (e.g., Krugman 1991, Ottaviano et al. 2002, Pflüger 2004, Picard and Tabuchi 2010). In almost all the dynamic models of the new economic geography, physical capital is neglected, and regional amenities do not play a significant role in determining land rent and population mobility. Determination of land values is also neglected in dynamic analysis of the literature. Although this approach is claimed to have "enabled researchers to gain further insights into the space economy and its transition" (Tabuchi 2014, p. 50), it is difficult to imagine any modern economy whose dynamics can be properly modelled neither with capital accumulation nor land value dynamics. In the literature of the new economic geography, as Tabuchi (2014, p. 50) observes, "The scopes of most of the theoretical studies published thus far have been limited to two regions in order for researchers to reach meaningful analytical results. The two-region NEG models tend to demonstrate that spatial distribution is dispersed in the early period (high trade costs or low manufacturing share) and agglomerated in one of the two regions in the late period (low trade costs or high manufacturing share). However, it is no doubt that the two-region NEG models are too simple to describe the spatial distribution of economic activities in real-world economies. Since there are only two regions, their geographical locations are necessarily symmetric, and thus diverse spatial distributions cannot occur." It is important to develop a model with any number of regions in order to properly address issues related to interregional growth and agglomeration. Many regions interact with each other in terms of trade and migration.

This study introduces endogenous amenity in explaining regional agglomeration. The role of amenities in spatial economic is well emphasized in the literature (e.g., Kanemoto 1980, Diamond and Tolley 1981, Glaeser et al. 2001, Chen et al. 2013). According to Chen et al. (2013), "Empirical evidence demonstrates a growing link between the presence of high-valued natural amenities – including pleasant climate and proximity to lakes, oceans, forests, and mountains – and higher rates of population and income growth in the U.S." There are different ideas about amenities and spatial economics, for instance, equilibrium ideas by Graves (1979) and Roback (1982), turnaround migration theory by Brown et al. (1997), life cycle studies by Clark and Hunter (1992), research on rural development by Deller et al. (2001). Zhang (1993b) first introduced spatial amenity into utility in a general equilibrium framework. Zhang (1998, 2018) introduced spatial amenity into a formal regional growth model. This paper is an extension of Zhang's recent multi-region growth model (Zhang, 2018). This study generalizes the previous paper mainly by introducing exogenous land values on basis of Zhang (2014). This paper is organized as follows. Section 2 defines the multi-region model with capital accumulation, economic structure, and land values. Section 3 identifies the differential equations which can be used to simulate the model, plots the motion of the model, demonstrates the existence of an equilibrium point, and proves the stability of the equilibrium point. Section 4 carries out comparative dynamic analysis. Section 5 concludes the study. The main analytical results of Section 3 are proved in the appendix.

2 UZAWA'S TWO-SECTOR GROWTH MODEL TO A MULTI-REGIONAL ECONOMY

This study assumes that land can be bought and sold in land markets without time delay and transaction costs. The model is influenced by many approaches in theoretical economics. Its core part with regard to production is following the neoclassical growth theory. Its trade pattern determination is based on the neoclassical trade theory with capital accumulation (Uzawa 1961, Oniki and Uzawa 1965, Sorger 2003). There is only one (durable) good in the national economy under consideration. Each region produces one goods and services. Commodities are traded without any barriers. We neglect transport costs. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. All savings volunteered by households are absorbed by firms. We require saving and investment to be equal at any point in time. The population N is homogenous. People are free to choose their residential location and people work and reside in the same region. Each region has fixed land L_j which is homogenous within each region. The assumption of zero transportation cost of commodities implies price equality for the commodity between regions. As amenity and land are immobile, wage rates and land rent may vary between regions. We use subscripts, i, s, to denote the industrial and services sectors, respectively. We measure prices in terms of the commodity and the price of the commodity be unity. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the jth region. The interest rate is equalized throughout the national economy, i.e., $r(t) = r_j(t)$. Let $F_{qj}(t)$ stand for the output levels of q's sector in region j at time t, $q = i, s$.

Behavior of producers. We assume that there are two productive factors, capital $K_{qj}(t)$ and labor $N_{qj}(t)$ at each point in time t. The production functions are specified as:

$$F_{qj}(t) = A_{qj}\, K_{qj}^{\alpha_{qj}}(t)\, N_{qj}^{\beta_{qj}}(t), q = i, s, j = 1, \ldots, J. \quad (1)$$

We use $p_j(t)$ to stand for region j's services price. As markets are competitive, labor and capital earn their marginal products. The production sector chooses the two variables, $K_{qj}(t)$ and $N_{qj}(t)$, to maximize its profit with the rate of interest and wage rates determined by markets.. The marginal conditions are:

$$r(t) + \delta_{kj} = \frac{\alpha_{ij}\, F_{ij}(t)}{K_{ij}(t)} = \frac{\alpha_{sj}\, p_j(t)\, F_{sj}(t)}{K_{sj}(t)},\ w_j(t) = \frac{\beta_{ji}\, F_{ij}(t)}{N_{ij}(t)} = \frac{\beta_{sj}\, p_j(t)\, F_{sj}(t)}{N_{sj}(t)}, \quad (2)$$

where $r(t)$ are the depreciation rate of physical capital in region j.

Choice between wealth and land. It is necessary to determine land ownership structure. Land may be owned by different agents under various institutions. This study assumes that land is owned by households. There are different approaches with regards to determination of land prices and rents. For instance, in the literature of urban economics two types of land distribution are often assumed. The one is the so-called absentee landlord. Under this assumption the landlords spend their land incomes outside the economic system. Another type, for instance as accepted in Kanemoto (1980), assumes that the urban government rents the land from the landowners at certain rent and sublets it to households at the market rent, using the net revenue to subsidize city residents equally. In some approaches (Iacoviello 2005, Iacoviello and Neri 2010) households are assumed to be credit constrained and these households use land or houses as collateral to finance consumption expenditures. These models with credit-constrained households are used to explain positive co-movements between house prices and consumption expenditures (see also, Campbell and Mankiw 1989, Liu et al. 2011). In our approach we assume that land can be sold and bought in free markets without any friction and transaction costs. Land use will not waste land and land cannot regenerate itself. Households own land and physical wealth. This study assumes that households can own land in any region. Let region j's land price be represented by $\bar{p}_j(t)$, and the land rent by $R_j(t)$. Consider now an investor with one unity of money. He can either invest in capital good thereby earning a profit equal to the net own-rate of return $r(t)$ or invest in land thereby earning a profit equal to the net own-rate of return $R_j(t)/\bar{p}_j(t)$. As we assume capital and land markets to be at competitive equilibrium at any point in time, two options must yield equal returns, i.e.

$$\frac{R_j(t)}{\bar{p}_j(t)} = r(t), j = 1, \dots, J. \tag{3}$$

This equation enables us to determine choice between owning land and wealth. This assumption is made under strict conditions. For instance, we neglect any transaction costs and any time needed for buying and selling land. Expectations on land are complicated. Equation (3) also implies perfect information and rational expectation.

Behavior of consumers. Consumers make decisions on choice of lot size, consumption levels of services and commodities as well as on how much to save and how much land to own. This study uses the approach to consumers' behavior proposed by Zhang (1993a). Let $\bar{k}_j(t)$ and $\bar{l}_{jq}(t)$ stand for respectively wealth and land held by the representative household in region j. From the land ownership, the interest payment of wealth and the wage income, the representative household of region j obtains the current income $y_j(t)$

$$y_j(t) = r(t)\bar{k}_j(t) + w_j(t) + \bar{r}_j(t), \tag{4}$$

where the land rent income is given by

$$\bar{r}_j(t) = \sum_{q=1}^{J} R_q(t)\bar{l}_{jq}(t), j = 1, \dots, J. \tag{5}$$

The total value of wealth and land held by the representative household in region j is

$$a_j(t) = \bar{k}_j(t) + v_j(t),$$

where $v_j(t)$ is the total land value held by the representative household in region j.

$$v_j(t) = \sum_{q=1}^{J} \bar{p}_j(t)\bar{l}_{jq}(t). \tag{7}$$

Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The household of region j can sell $a_j(t)$ to purchase goods and to save. The disposable income $\hat{y}_j(t)$ is the sum of the current income and the value of wealth held by the representative household. That is

$$\hat{y}_j(t) = y_j(t) + a_j(t). \tag{8}$$

The disposable income which is the sum of the current income and the value of the total wealth held by the household is used for saving and consumption. The consumer of region j distributes the total available budget between housing $l_j(t)$, saving $s_j(t)$, consumption of goods $c_{ij}(t)$, and consumption of services $c_{sj}(t)$. The budget constraints are

$$c_{ij}(t) + p_j(t)c_{sj}(t) + R_j(t)l_j(t) + s_j(t) = \hat{y}_j(t). \tag{9}$$

We specify utility functions $U_j(t)$ as follows

$$U_j(t) = \theta_j(t)\, l_j^{\eta_0}(t)\, c_{ij}^{\xi_0}(t)\, c_{sj}^{\gamma_0}(t)\, s_j^{\lambda_0}(t), \eta_0, \xi_0, \gamma_0, \lambda_0 > 0, \tag{10}$$

in which η_0, ξ_0, γ_0, and λ_0 are the household's elasticity of utility with regards to lot size, commodity, services, and savings. We call η_0, ξ_0, γ_0, and λ_0 propensities to consume lot size, goods, and services, to hold wealth (save), respectively. In (10), $\theta_j(t)$ is called region j's amenity level. Amenities are affected by infrastructures, regional cultures and climates. Amenity θ_j is assumed to be affected by the population in the following way:

$$\theta_j(t) = \bar{\theta}_j\, N_j^d(t), j = 1, \dots, J, \tag{11}$$

where $\bar{\theta}_j (> 0)$, d are parameters and $N_j(t)$ is region j's population. We don't specify sign of d as the population may have either positive or negative effects on regional attractiveness. As Chen et al. (2013, p. 269) observe: "The presence of both positive and negative population externalities suggests that the steady state (or competitive) pattern may differ from an optimal pattern in which all the external benefits and costs of households' migration decisions are internalized." We will examine effects of changes in amenity parameters on not only steady state but also transitory processes of the economic system.

The first-order conditions of maximizing the utility subject to (9) are

$$l_j(t)\,R_j(t) = \eta\,\hat{y}_j(t), c_{ij}(t) = \xi\,\hat{y}_j(t), p_j(t)\,c_{sj}(t) = \gamma\,\hat{y}_j(t), s_j(t) = \lambda\,\hat{y}_j(t), \quad (12)$$

where

$$\eta \equiv \eta_0\,\rho, \xi \equiv \xi_0\,\rho, \gamma \equiv \gamma_0\,\rho\,, \lambda \equiv \lambda_0\,\rho, \rho \equiv \frac{1}{\eta_0 + \xi_0 + \gamma_0 + \lambda_0}.$$

Wealth accumulation. According to the definitions of $s_j(t)$, the wealth change of the representative household in region j is given by

$$\dot{a}_j(t) = s_j(t) - a_j(t). \quad (13)$$

Equalization of utility levels between regions. As households are assumed to be freely mobile between the regions, the utility level of people should be equal, irrespective of in which region they live, i.e.

$$U_j(t) = U_q(t), j, q = 1, \ldots, J. \quad (14)$$

Possible costs for migration are omitted. It should be remarked that wage equalization between regions is often used as the equilibrium mechanism of population mobility over space. This study assumes that households obtain the same level of utility in different regions as the equilibrium mechanism of population distribution between regions.

The demand and supply balance for services. A region's supply of services is consumed by the region

$$c_{sj}(t)\,N_j(t) = F_{sj}(t), j = 1, \ldots, J. \quad (15)$$

Capital being fully used. The total capital stocks $K(t)$ employed by the production sectors is equal to the total wealth owned by the households of all the regions. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{k}_j(t)\ N_j(t), \tag{16}$$

in which

$$K_j(t) \equiv K_{ji}(t) + K_{js}(t).$$

Full employment of labor. The assumption that labor force is fully employed implies:

$$N_{ij}(t) + N_{sj}(t) = N_j(t). \tag{17}$$

The national population balance.

$$\sum_{j=1}^{J} N_j(t) = N, j = 1, \dots, J. \tag{18}$$

Land is fully used. The assumption that land is fully employed implies

$$l_j(t)\ N_j(t) = L_j, j = 1, \dots, J. \tag{19}$$

Land ownership distribution. A region's land is fully owned by the population

$$\sum_{q=1}^{J} \bar{l}_{qj}(t)\ N_q(t) = L_j, j = 1, \dots, J. \tag{20}$$

We have thus built the model. The model describes dynamic interactions among capital accumulation, regional capital and labor distribution, and land values and rents in a national economy in which all the markets are perfectly competitive and product, capital and labor are freely mobile.

3 Simulating the Model

The economic system is complicated as it consists of many regions and each region has many variables. Although the dynamic system is highly dimensional and nonlinear, we can know its behavior for given functional forms and specified parameter values by simulation. The rest of the study simulates the model. Before simulating the model, we provide a computational program which enables anyone with portable computer to see the motion of the system. We show that the dynamics of the national economy can be expressed as $J+1$dimensional differential equations. First, we introduce a variable $z_1(t)$

$$z_1(t) \equiv \frac{r(t)+\delta_{k1}}{w_1(t)}.$$

Lemma. The motion of the national economy is given by the following $J+1$ differential equations with $z_1(t)$ and $\left(a_j(t)\right)$ as variables

$$\dot{a}_j(t) = \Phi_j\left(z_1(t), a_j(t)\right), j = 1, \ldots, J,$$
$$\dot{z}_1(t) = \Phi_0\left(z_1(t), \left(a_j(t)\right)\right), \tag{21}$$

where Φ_j and Φ_0 are functions of $z_1(t)$ and $\left(a_j(t)\right)$ defined in the appendix. For any given positive values of $z_1(t)$ and $\left(a_j(t)\right)$ at any point in time, the other variables are uniquely determined by the following procedure:

$r(t)$ by (A2) → $w_j(t)$ by (A4) → $\hat{y}_j(t)$ by (A12) → $p_j(t)$ by (A5) → $N_1(t)$ by (A9) → $N_j(t)$ by (A10) → $R_j(t)$ by (A13) → $l_j(t) = L_j/N_j(t)$ → $\bar{p}_j(t)$ by (A13) →$N_{js}(t)$ by (A14) → $N_{ji}(t)$ by (A15) → $c_{ji}(t)$, $c_{ji}(t)$, and $s_j(t)$ by (12) → $K_{jq}(t)$ by (A1) → $F_{jq}(t)$ by definitions → $K_j(t) = K_{ji}(t) + K_{js}(t)$ → $K(t)$ by (16) → $Y_j(t) = F_{ji}(t) + p_j(t)F_{js}(t)$ → $Y(t) = \sum_j Y_j(t)$.

Our dynamic equations are highly dimensional and nonlinear. It is built on microeconomic foundation with regional characteristics. The lemma provides a computational procedure for following the motion of the economic system with any number of regions. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model for a 3-region economy. We specify parameter values as follows:

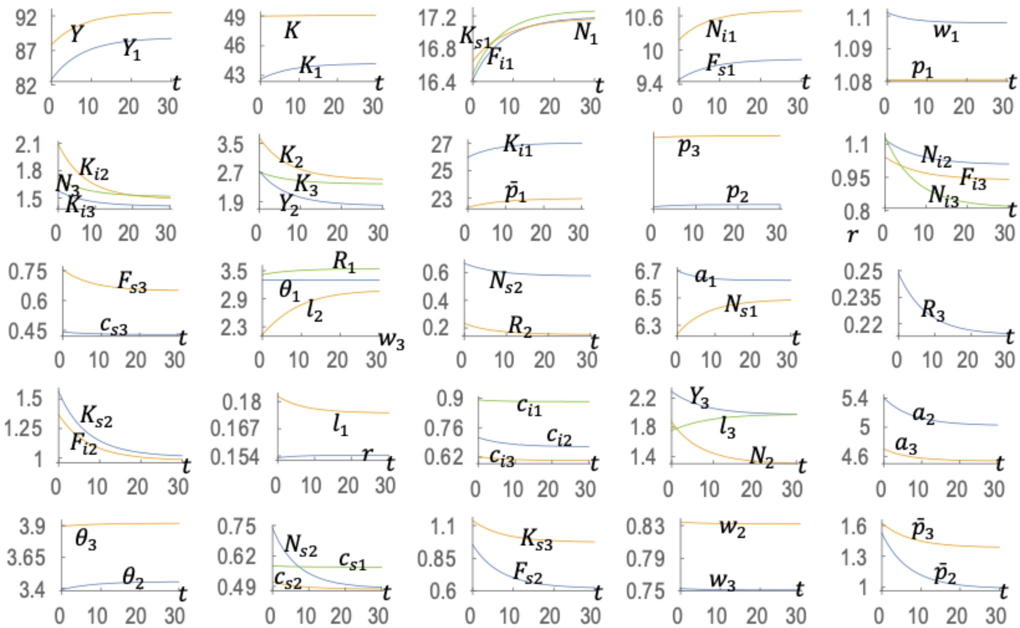
$$N = 20, \lambda_0 = 0.75, \xi_0 = 0.1, \eta_0 = 0.07, \gamma_0 = 0.07, d = -0.05,$$
$$\begin{pmatrix} A_{i1} \\ A_{i2} \\ A_{i3} \end{pmatrix} = \begin{pmatrix} 1.2 \\ 1 \\ 0.95 \end{pmatrix}, \begin{pmatrix} A_{s1} \\ A_{s2} \\ A_{s3} \end{pmatrix} = \begin{pmatrix} 1.1 \\ 1 \\ 0.95 \end{pmatrix}, \begin{pmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \alpha_{i3} \end{pmatrix} = \begin{pmatrix} 0.32 \\ 0.31 \\ 0.3 \end{pmatrix}, \begin{pmatrix} \alpha_{s1} \\ \alpha_{s2} \\ \alpha_{s3} \end{pmatrix} = \begin{pmatrix} 0.33 \\ 0.34 \\ 0.325 \end{pmatrix}, \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \\ \bar{\theta}_3 \end{pmatrix} = \begin{pmatrix} 3.8 \\ 3.5 \\ 4 \end{pmatrix}, \begin{pmatrix} \delta_{k1} \\ \delta_{k2} \\ \delta_{k3} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.05 \\ 0.06 \end{pmatrix}. \tag{22}$$

Region 1's levels of productivity of the two sectors are highest; region 2's levels are the next; and region 3's levels of productivity of the two sectors are lowest. We specify values of α_{jk} close to 0.3. With regard to the technological parameters, for illustration what are important in our interregional study are their relative values. The presumed productivity differences between the regions are not very large. The specified values of the land sizes, the preference parameters and the population will not affect our main concerns about interactions between the regions. We specify the initial conditions as follows

$$z_1(0) = 0.1845, a_1(0) = 6.7, a_2(0) = 5.4, a_3(0) = 4.7.$$

Figure 1: The motion of the economic system



Source: processed by author.

The motion of the variables is plotted in Figure 1. The national output rises over time till they arrive at the equilibrium level. The national wealth and the rate of interest are slightly changed over time. Region 1's total output and two sectors' output levels rise, the other two regions' total output and two sectors' output levels are reduced. Some people migrate from regions 2 and 3 to region 1. Region 1's amenity slightly falls, and the other two regions' amenity levels are improved. The service prices in the three regions are slightly augmented. The wage rates in the three regions are lowered over time. Region 1's land value rises and the other two regions' land values fall. The lot size in region 1 falls and the lot sizes in the other two regions rise.

It is straightforward to confirm that all the variables become stationary in the long term. This implies the existence of an equilibrium point. We list the equilibrium values in equation 16.

$$Y = 92.5, K = 49.5, r = 0.153,$$

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 88.71 \\ 1.79 \\ 1.97 \end{pmatrix}, \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 17.2 \\ 1.29 \\ 1.51 \end{pmatrix}, \begin{pmatrix} F_{i1} \\ F_{i2} \\ F_{i3} \end{pmatrix} = \begin{pmatrix} 17.41 \\ 0.99 \\ 1.01 \end{pmatrix}, \begin{pmatrix} F_{s1} \\ F_{s2} \\ F_{s3} \end{pmatrix} = \begin{pmatrix} 9.8 \\ 0.62 \\ 0.64 \end{pmatrix},$$

$$\begin{pmatrix} N_{i1} \\ N_{i2} \\ N_{i3} \end{pmatrix} = \begin{pmatrix} 10.75 \\ 0.82 \\ 0.94 \end{pmatrix}, \begin{pmatrix} N_{s1} \\ N_{s2} \\ N_{s3} \end{pmatrix} = \begin{pmatrix} 6.45 \\ 0.48 \\ 0.57 \end{pmatrix}, \begin{pmatrix} K_{i1} \\ K_{i2} \\ K_{i3} \end{pmatrix} = \begin{pmatrix} 27.41 \\ 1.51 \\ 1.42 \end{pmatrix}, \begin{pmatrix} K_{s1} \\ K_{s2} \\ K_{s3} \end{pmatrix} = \begin{pmatrix} 17.2 \\ 1 \\ 0.97 \end{pmatrix},$$

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 1.08 \\ 0.98 \\ 0.99 \end{pmatrix}, \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1.1 \\ 0.83 \\ 0.75 \end{pmatrix}, \begin{pmatrix} \bar{p}_1 \\ \bar{p}_2 \\ \bar{p}_3 \end{pmatrix} = \begin{pmatrix} 23.05 \\ 0.99 \\ 1.39 \end{pmatrix}, \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \begin{pmatrix} 3.53 \\ 0.15 \\ 0.21 \end{pmatrix}, \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} 3.3 \\ 3.46 \\ 3.91 \end{pmatrix},$$

$$\begin{pmatrix} a_1 \\ a_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} 6.6 \\ 5 \\ 4.52 \end{pmatrix}, \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} 0.18 \\ 3.09 \\ 1.99 \end{pmatrix}, \begin{pmatrix} c_{i1} \\ c_{i2} \\ c_{i3} \end{pmatrix} = \begin{pmatrix} 0.88 \\ 0.67 \\ 0.6 \end{pmatrix}, \begin{pmatrix} c_{s1} \\ c_{s2} \\ c_{s3} \end{pmatrix} = \begin{pmatrix} 0.57 \\ 0.48 \\ 0.43 \end{pmatrix}. \quad (23)$$

It is straightforward to calculate the eigenvalues as follows

$$\{-0.1587, -0.1264, -0.1263, 0\}.$$

The three eigenvalues are real and negative, and one is zero. As shown in the appendix the dynamics are three dimensional. The unique equilibrium is locally stable. This guarantees the validity of exercising comparative dynamic analysis.

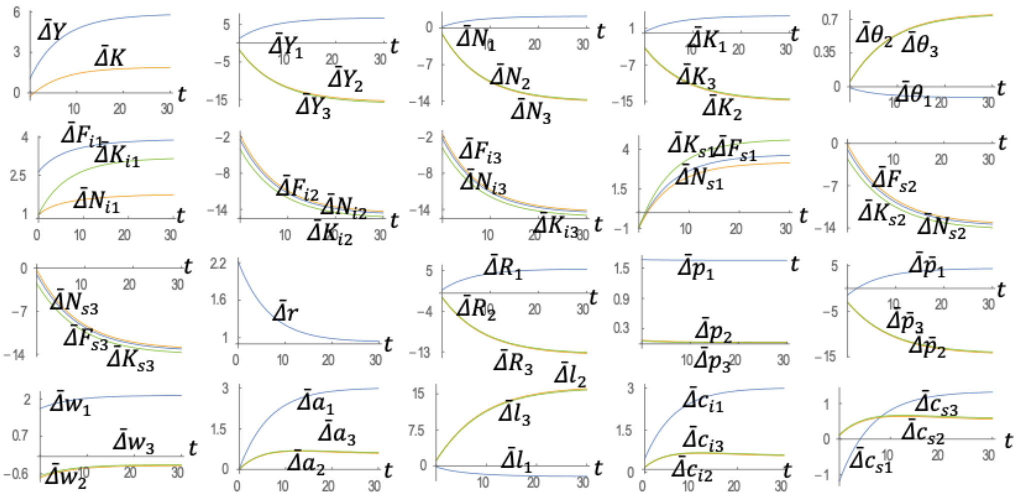
4 Comparative dynamic analysis

We simulated the motion of the national economy under (15). We now study how the economic system reacts to changes, for instance, in the total productivities and the preference. As the lemma gives a computational procedure to calibrate the motion of all the variables, it is straightforward to examine transitory processes towards the long-term equilibrium for change in any parameter. In the rest of this study we use $\bar{\Delta} x_j(t)$ to stand for the change rate of the variable $x_j(t)$ in percentage due to changes in a parameter value.

4.1 The total factor productivity of region 1's industrial sector being enhanced

We first study the effects of a technological improvement in region 1's industrial sector. The technological progress is specified as follows: $\boldsymbol{A_{i2}}: \mathbf{1.2} \Rightarrow \mathbf{1.22}$. The simulation result is plotted in Figure 2. We see that the national output and national capital stock are augmented. This occurs in association with migration from the other two regions to region 1. As the value of $\boldsymbol{A_{i1}}$ before being increased is higher than the total factor productivity in any other region, this implies that if all the other conditions being the same, then migration from the other regions to region 1 tend to increase the national output. Region 1's lot size falls and land rent rises in tandem with rising population. The other two regions' lot sizes are increased, and land rents are reduced. Wealth per capita in all the regions is increased. Region 1's service price, wage rate, and consumption of the industrial goods are enhanced, while these variables in the other two regions are reduced. The per capita service consumption level in the region 1 falls initially and then rises. This happens as the price rises rapidly but the rises in the wage and wealth take a longer time before the net impact on service consumption becomes positive. The per capita service consumption levels in the other two regions are augmented. Region 1's land value falls initially and rises in the long term. The other regions' land values are reduced in association with falling population. The total output level of region 1 and the capital stocks employed by the regions are increased. The total output levels of regions 2 and 3 and the capital stocks employed by the regions are lowered. Region 1's amenity falls and the other two regions' amenity levels are enhanced. The output level and capital and labor inputs of region 1's industrial sector are increased. The output level and capital and labor inputs of region 1's service sector are lowered initially and increased in the long term. The output levels and capital and labor inputs of the other two region's industrial and service sectors are all reduced. It is interesting to note that the rate of interest rises as the total factor productivity is increased. In the standard neoclassical growth theory where land and land values are omitted, a rise in the productivity tends to reduce the rate of interest. As the land values in some regions fall in our interregional economy, people tend to hold physical wealth which tends to enhance the rate of interest. Indeed, the rise in rate of interest is a consequence of interactions of multiple forces. Another insight we obtain from this analysis is about dynamics of wage disparities over time between regions. Wage disparities are caused by many factors, such as spatial differences in education opportunities, knowledge diffusion, skill composition of the workforce, local interactions, discrimination, as well as non-human endowments (for instance, Glaeser and Maré, 2001; Duranton and Monastiriotis, 2002; Combes *et al* 2003; Rey and Janikas, 2005). From our simulation result, we see that the wage disparity is strongly affected by change in technology. This also hints that if technological differences between regions are not large, wage rates may tend to converge if the other factors weakly affect the differences. It can be seen that the rise of productivity in the technologically advanced region causes further gaps between the region and the other regions in terms of wage rate, regional output, amenity levels, and wage rates.

Figure 2: A rise in the total factor productivity of region 1's industrial sector

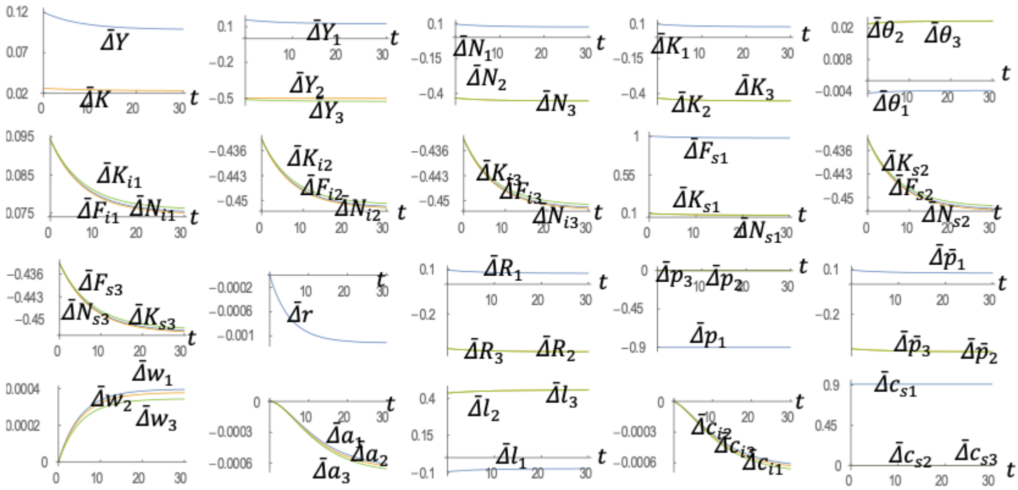


Source: processed by author.

4.2 A rise in the total factor productivity of region 1's service sector

We now examine the effects of the following rise in the total factor productivity region 1's service sector: $A_{s1}: 1.1 \Rightarrow 1.11$. The simulation result is plotted in Figure 3. The national output and wealth are augmented by the productivity improvement. This happens partly as people migrate from the less advanced regions to the technologically more advanced region, resulting in the enhancement in the national income and wealth. Region 1 attracts more people from the other two regions. Region 1's total output level is increased and the other two regions' output levels are lowered. Region 1's amenity is deteriorated and the other two regions' amenities are improved. The migration results in falls in region 1's lot size and rises in land rent; the other two regions' lot sizes are increased and land rents are reduced. Region 1's land value is enhanced and the other two regions' land values are reduced. The rate of interest is increased. Wage rates, per capita wealth levels, and per capita consumption levels of industrial goods are slightly affected. Region 1's service price falls in association with the service sector improvement in productivity. The other two regions' service prices are slightly affected.

Figure 3: A rise in the total factor productivity of region 1's service sector

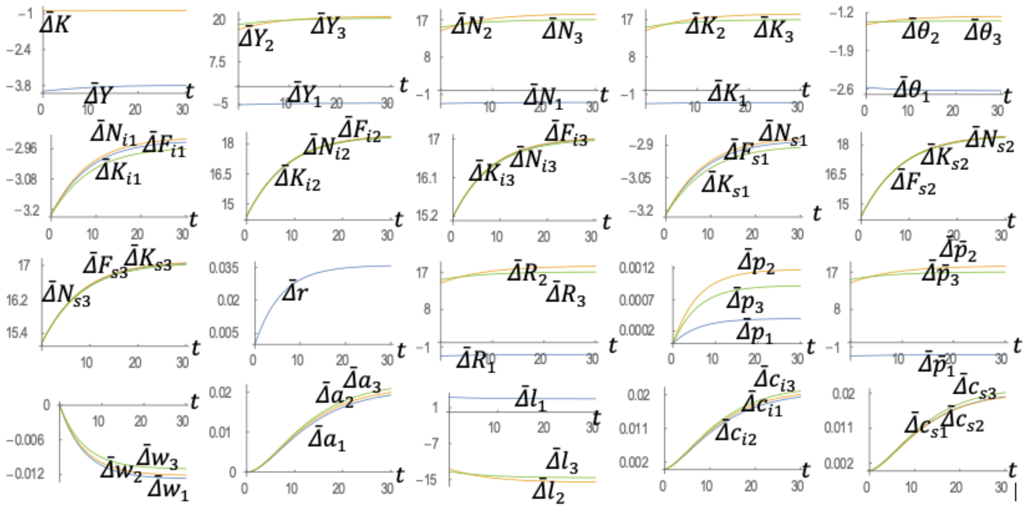


Source: processed by author.

4.3 The population having stronger negative impact on amenity

We now examine what will happen to the economic system if the population has stronger negative impact on amenity as follows: $d: -0.05 \Rightarrow -0.06$. The simulation result is plotted in Figure 4. The national output and wealth are reduced. As the population has stronger negative impact on amenity, amenities tends to be deteriorated. The population is redistributed as amenities are affected. The regions' amenities are deteriorated. Region 1's amenity is deteriorated more than the other two regions'. Region 1 loses some worker forces and the other two regions get more labor forces. Region 1's total output falls and the other two regions' total output levels are augmented. The national output and total capital stock are reduced. This occurs mainly because people migrate from the productive region to the less productive regions. The migration results in rises in region 1's lot size and falls in land rent; the other two regions' lot sizes are reduced and land rents are increased. Region 1's land value is lowered and the other two regions' land values are enhanced. The rate of interest is slightly increased. Wage rates, prices of services, per capita wealth levels, and per capita consumption levels of industrial goods and services are slightly affected.

Figure 4: The population having stronger negative impact on amenity

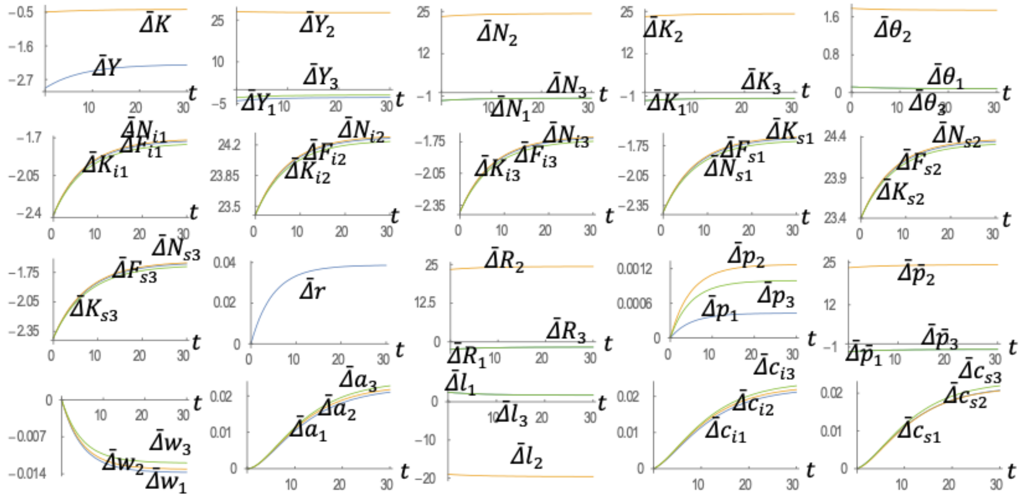


Source: processed by author.

4.4 Region 2's amenity parameter being augmented

We now study what happens in the economic system if the system experiences the following rise in region 2's amenity parameter: $\bar{\theta}_2: 3.5 \Rightarrow 3.7$. The simulation result is plotted in Figure 5. As pointed out by Chen (2013, p. 256), "number of empirical studies have demonstrated the positive association between rural growth and natural amenities (e.g., McGranahan 1999, Kim et al. 2005), none have examined the pattern of relative population distribution across amenity-based areas nor sought to develop a theoretical model of amenity-led migration that explains this distribution." As our model is a general equilibrium one, we can explain how amenities interaction with population distribution. Region 2 attracts more people and region 1 loses some labor force. Region 3's labor and capital stocks employed are slightly affected. The national output and capital stocks are reduced. The rate of interest rises and the wage rates in all the regions slightly fall. Region 2's total output is increased and other two regions' total output levels are reduced. Region 2's land rent and value are increased as its amenity is improved. It should be noted that our approach on regional housing markets can be related to hedonic price modelling (e.g., Rosen 1974, Helbich et al. 2014). The approach is influenced by Lancaster's idea that it is a good's characteristics that creates utility. When we apply this idea to housing market which are tied with environment and land, it implies that amenity should have effects on housing prices (Dubin 1992, Can and Megbolugbe 1997, Sheppard 1997, Malpezzi 2003, McMillen 2010, Ahlfeldt 2011). Our model shows how land values and rents are related to differences in productivities and amenities between regions.

Figure 5: Region 2's amenity parameter being augmented

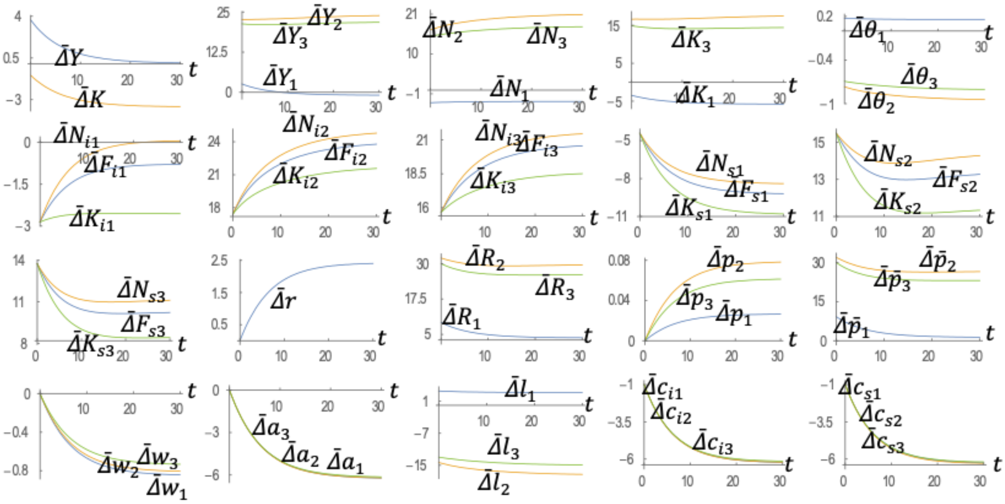


Source: processed by author.

4.5 The propensity to consume housing being increased

We now study the effects of the following rise in the population's propensity to consume housing: $\eta_0 : 0.07 \Rightarrow 0.08$. The simulation result is plotted in Figure 6. The national output is increased and the national physical capital is reduced. The rate of interest rises, and the wage rates fall slightly in all the regions. Some of region 1's labor force migrates to regions 2 and 3. The land rents and values in all the regions are increased. Although region 1 lose some of its population, its land rent and value rise as well. Region 1's total output rises initially and falls in the long term. The other two regions' output levels are enhanced. Region 1's lot size is increased and the other two regions' lot sizes are reduced. Region 1's amenity is improved and the other two regions' amenity levels are deteriorated. The consumption levels and wealth levels per household are reduced in all the regions.

Figure 6: The propensity to consume housing being increased

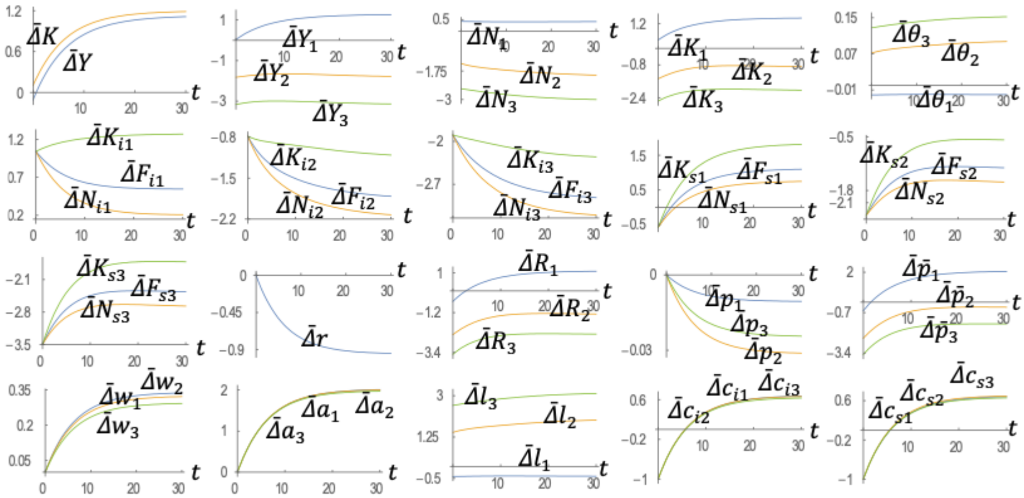


Source: processed by author.

4.6 A rise in the propensity to save

Effects of saving propensity changes are different in different theories. In the Keynesian economic theory savings tend to reduced national income. The neoclassical growth theory argues the opposite effect. As only a few growth models with space take account of endogenous savings, regional growth theory has not much to say on how a change in the propensity to save can affect spatial agglomeration and regional economic growth. We now allow the propensity to save to be changed as follows: $\lambda_0: 0.75 \Rightarrow 0.76$. The simulation result is plotted in Figure 7. The national output and wealth are increased in association with falling rate of interest. The change in the propensity to save has a strong impact on regional disparity and population distribution. As the economy has more capital, region 1 attracts more people from the other two regions. This results in enlarged differences between region 1 and the other two regions. Region 1's regional income is increased, while the other two regions" regional income levels are reduced. Region 1 employs more capital while the other two regions us less capital. The two sectors' output levels in region 1 are increased while the two sectors' output levels in the other two regions are reduced. The wage rates and wealth levels per household in all the regions are increased. Region 1's amenity is deteriorated, while the amenity levels in the other regions are improved. The consumption levels of both industrial goods and services in all the regions are reduced initially and increased in the long term. Region 1's land value is increased and the other two regions' land values are reduced.

Figure 7: A rise in the propensity to save



Source: processed by author.

5 Conclusions

This paper introduced endogenous land values into the multi-regional growth model proposed by Zhang (2018). The economy consists of any number of regions and each region has two sectors. Land values, land rents, capital accumulation and regional amenities are endogenous. The economy is built under assumptions of profit maximization, utility maximization, and perfect competition. We used the utility function proposed by Zhang (1993) to determine saving and consumption. The dynamics of J-regional economy is controlled by $J + 1$ differential equations. We simulated the model with a 3-region model and demonstrated the existence of a unique equilibrium point. We also conducted comparative analysis to provide some insights. As the model is structurally general, it is possible to deal with various national as well as regional growth and environment issues. It is straightforward to analyze behavior of the model with other forms of production or utility functions. Households should be heterogeneous. Also issues related to tax competition between regions have caused great attention in economic geography (for instance, Borck and Pflüger, 2006). There are different studies on regional economic growth with endogenous knowledge (Banerjee and Jarmuzek, 2010). Issues related to land, for instance conversion of land to different uses and land as input factors to different uses, and imperfect land markets, are referred to Marjit and Kar (2019).

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APPENDIX 1: PROVING THE LEMMA

We now prove the procedure in the lemma. First, from equations 2 we obtain:

$$z_j \equiv \frac{r+\delta_k}{w_j} = \frac{\bar{b}_j N_{ij}}{K_{ij}} = \frac{b_j N_{sj}}{K_{sj}}, \tag{A1}$$

where

$$\bar{b}_j \equiv \frac{\alpha_{ij}}{\beta_{ij}}, b_j \equiv \frac{\alpha_{sj}}{\beta_{sj}}.$$

Insert $z_j/a_j \equiv N_{ij}/K_{ij}$ in $r+\delta_{kj} = \alpha_{ij}F_{ij}/K_{ij}$ from equation 2

$$r(z_j) = \frac{\alpha_{ij}\, A_{ij}}{\bar{b}_j^{\beta_{ij}}} z_j^{\beta_{ij}} - \delta_{kj}, j = 1,\dots,J. \tag{A2}$$

From (A2) we get

$$z_j(z_1) = \bar{b}_j \left(\frac{r+\delta_{kj}}{\alpha_{ij}\, A_{ij}}\right)^{1/\beta_{ji}}, j = 2,\dots,J. \tag{A3}$$

From (A1) and (A2), we have

$$w_j(z_1) = \frac{r+\delta_k}{z_j}. \tag{A4}$$

From $z_j = b_j N_{sj}/K_{sj}$ and (2), we have

$$p_j(z_1) = \frac{b_j^{\beta_{sj}}(r+\delta_k)}{\alpha_{sj}\, A_{sj}\, z_j^{\beta_{sj}}}. \tag{A5}$$

From (12) and (15) we have

$$\gamma_j\, \hat{y}_j\, N_j = p_j\, F_{sj}, j = 1,\dots,J. \tag{A6}$$

Insert (2) in (A6)

$$\gamma\ \hat{y}_j\ N_j = \frac{w_j\ N_{sj}}{\beta_{sj}}, j = 1, \ldots, J. \tag{A7}$$

Substitute $l_j = L_j/N_j$, (9), and (10) into (8)

$$U_j = \frac{\bar{\theta}_j\ N_j^{d-\eta_0}\ L_j^{\eta_0}}{p_j^{\gamma_0}} \xi^{\xi_0}\gamma^{\gamma_0}\lambda^{\lambda_0}\hat{y}_j^{\omega}, \tag{A8}$$

where $\omega \equiv \xi_0 + \gamma_0 + \lambda_0$. Apply $U_j = U_q$ to (A8)

$$N_j = \Lambda_j\ N_1, \tag{A9}$$

where

$$\Lambda_j\left(z_1, \hat{y}_j\right) \equiv \left(\frac{\bar{\theta}_1\ L_1^{\eta_0}\ p_j^{\gamma_0}}{\bar{\theta}_j\ L_j^{\eta_0}\ p_1^{\gamma_0}}\right)^{\frac{1}{(d-\eta_0)}} \left(\frac{\hat{y}_1}{\hat{y}_j}\right)^{\frac{\omega}{d-\eta_0}}.$$

Insert (A9) in (14)

$$N_1\left(z_1, \left(\hat{y}_j\right)\right) = \frac{N}{\sum_{j=1}^{J} \Lambda_j}, \Lambda_j = 1. \tag{A10}$$

With (A9) and (A10) we determine the population distribution as functions of z_1 and $\left(\hat{y}_j\right)$. Insert (3) in (5)

$$\bar{r}_j = r \sum_{q=1}^{J} \bar{p}_j\ \bar{l}_{jq} = r\ v_j, j = 1, \ldots, J, \tag{A11}$$

where we also use (7). By (4), (8) and (A11), we have

$$\hat{y}_j\left(z_1, a_j\right) = (1 + r)a_j + w_j. \tag{A12}$$

By $l_j R_j = \eta \hat{y}_j$ and $l_j N_j = L_j$, we have

$$R_j\left(z_1, \left(a_j\right)\right) = \frac{\eta\ \hat{y}_j\ N_j}{L_j}. \tag{A13}$$

From (A7) we have

$$N_{sj}(z_1, a_j) = \frac{\gamma\ \beta_{sj}\ \hat{y}_j\ N_j}{w_j}, j = 1, \ldots, J. \tag{A14}$$

From $N_{ji} + N_{js} = N_j$ and (A14), we have

$$N_{ij}\left(z_1, (a_j)\right) = N_j - N_{sj}, j = 1, \ldots, J. \tag{A15}$$

From (A1) we have

$$K_{ij}\left(z_1, (a_j)\right) = \frac{\bar{b}_j\ N_{ij}}{z_j}, K_{sj}\left(z_1, (a_j)\right) = \frac{b_j\ N_{sj}}{z_j}. \tag{A16}$$

From (A16) and (16) we have

$$K_j\left(z_1, (a_j)\right) = K_{ji}\left(z_1, (a_j)\right) + K_{js}\left(z_1, (a_j)\right), K\left(z_1, (a_j)\right) = \sum_{j=1}^{J} K_j\left(z_1, (a_j)\right). \tag{A17}$$

From (14), we have

$$\sum_{j=1}^{J} \bar{k}_j\ N_j = K. \tag{A18}$$

Multiply (20) by $\bar{p}_j$

$$\sum_{q=1}^{J} \bar{p}_j \bar{l}_{qj} N_q = \bar{p}_j L_j, j = 1, \ldots, J. \tag{A19}$$

From (A19), we have:

$$\sum_{j=1}^{J} \sum_{q=1}^{J} \bar{p}_j \bar{l}_{qj} N_q = \sum_{j=1}^{J} \bar{p}_j L_j. \tag{A20}$$

Add (A18) and (A20)

$$\Psi\left(z_1,(a_j)\right) \equiv \sum_{j=1}^{J} a_j N_j - K - \frac{1}{r}\sum_{j=1}^{J} R_j L_j = 0, \qquad (A21)$$

where we also use (3). Substitute $s_j = \lambda_j \hat{y}_j$ into (10)

$$\dot{a}_j = \Phi_j\left(z_1,(a_q)\right) \equiv \lambda_j \hat{y}_j - a_j. \qquad (A22)$$

Taking derivatives of (A21) with respect to t yields

$$\dot{z}_1 = -\left(\sum_{j=1}^{J} \Phi_j \frac{\partial \Psi}{\partial a_j}\right)\left(\frac{\partial \Psi}{\partial z_1}\right)^{-1}, \qquad (A23)$$

where we use (A21). We don’t give expressions as they are tedious. Following the procedure in the lemma we describe the dynamics of the whole system.